+For Schools within our Collaboration:


SUTTON ROAD PRIMARY SCHOOL AND NURSERY


Equality Statement
At our collaboration of schools, we are committed to ensuring equality of education and opportunity for all pupils, staff, parents and carers receiving services from the school, irrespective of race, gender, disability, faith or religion or socio-economic background. We aim to develop a culture of inclusion and diversity in which all those connected to the school feel proud of their identity and able to participate fully in school life.
The achievement of pupils will be monitored by race, gender and disability and we will use this data to support pupils, raise standards and ensure
inclusive teaching. We will tackle discrimination by the positive promotion of equality, challenging bullying and stereotypes and creating an environment which champions respect for all. At our schools, we believe that diversity is a strength, which should be respected and celebrated by all those who learn, teach and visit here.
This booklet is designed to show the progression in calculation strategies for each of the four operations: addition, subtraction, multiplication and division. For each operation there are stages that children need to work through and build upon their basic skills. This should be used to bridge gaps within calculation processes and move through the stages at a speed appropriate to each year group.

## CALCULATION POLICY

This policy lays out the expectations for both mental and written calculations for the 4 number operations and has been created to support the teaching of a mastery approach to mathematics. This is underpinned by the use of models and images that support conceptual understanding and this policy promotes a range of representations to be used across the primary years. Mathematical understanding is developed through use of representations that are first of all concrete (e.g. Dienes apparatus and place value counters), and then pictorial (e.g. bar models) to then facilitate abstract working (e.g. standard written methods).This policy is a guide through an appropriate progression of representations and if at any point a pupil is struggling with the abstract, they should revert to familiar pictorial and/or concrete materials/representations as appropriate.

Although this policy sets out the main methods of mental and written calculations to be taught, it has been appended with a list of recommendations and effective practice teaching ideas aimed at informing and enhancing teaching across all the primary phases. Many of these ideas come from the DFE Mathematics guidance: key stages 1 and 2 (published June 2020) and the NCETM's Calculation Guidance document (published October 2015) and the which is intended to sit alongside a school's calculation policy.

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabulary | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| EYFS | If available, Numicon shapes are introduced straight away and can be used to : <br> Identify 1 more/less <br> Combine pieces to add <br> Find number bonds <br> Add without counting <br> Subitise/recognise patterns to support addition for example arrange objects as you would see them on a dice. <br> Adding with a tens frame for example we know if a tens frame is full this is 10 , one line equals 5 , one less than a line equals 4 etc. | 6+4 <br> 7+3 <br> 8+2 <br> 9+1 <br> Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). <br> Counting on using number lines using cubes or Numicon. | Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. <br> A bar model which encourages the children to count on, rather than count all. <br> 000000 <br> Children can use bead strings practically or colouring in different sums. For example: $4+3=7$ | $4+3=7$ <br> Four is a part, 3 is a part and the whole is seven. $\begin{aligned} & 7=4+3 \\ & 7=3+4 \end{aligned}$ <br> The abstract number line: What is 2 more than 4 ? What is the sum of 2 and 4 ? What is the total of 4 and 2? $4+2$ | Tens <br> Ones <br> Units <br> Add <br> More <br> And <br> Make <br> Sum <br> Total <br> Altogether <br> Double <br> One more <br> two more <br> ten more <br> Add five <br> more. <br> How many more to make ....? <br> How many more is ... than ...? | 100 square <br> Number lines <br> Number tracks <br> Bead strings <br> Tens Frame <br> Numicon <br> Place Value Counters <br> Base ten (Dienes) |

## Year One Addition




## Year Two Addition

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabulary | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Year } \\ 2 \end{gathered}$ | Use counting sticks, counting on, fingers or songs to add together 2 small amounts mentally. <br> Add a single-digit number to a two-digit number, including crossing the tens boundary, e.g. $23+5$, then $28+5$ <br> Add a multiple of 10 to any twodigit number, e.g. $27+60$ add two two-digit numbers <br> Add near doubles, <br> $7+8=15$ e.g. $13+$ <br> $7+7+1$ <br> $14,39+40$ <br> $14+1=15$ |  |  |  | Add <br> Sum <br> More than <br> Total <br> Altogether <br> Plus <br> Digit <br> Partition into tens and ones/units | 100 square <br> Number lines <br> Number tracks <br> Bead strings <br> Tens Frame <br> Numicon <br> Place Value Counters <br> Base ten <br> (Dienes) <br> Arrow Cards |



|  |  |  |  | $21+10=$ <br> 21 <br> Record the calculation <br> vertically adding the <br> column of ones then <br> the column of tens. |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

## Year 3 Addition

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabular y | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Year } \\ 3 \end{gathered}$ | Use number bonds to 20 and links to bonds of multiples of 10 to 100, complements to 100 e.g. $45+55$ $=100$ <br> Practise solving varied addition questions mentally with two-digit numbers, the answers could exceed 100. <br> Add numbers mentally, including: <br> - a three-digit number and ones <br> - a three-digit number and tens <br> - a three-digit number and hundreds <br> Recall number bonds to 20 fluently and derive and use related facts to 100 <br> Partition numbers in different ways Eg: $62=60+2,50+12,40+22$ etc <br> Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot. <br> Apply mental strategies to written methods: | 2-digit + 2-digit crossing 10s (into 100) <br> See Y2 and now crossing 100s and bridging/carrying $\begin{aligned} & 86+48=134 \\ & 1010 \\ & 10 \\ & 10 \\ & 10 \\ & 10 \\ & 10 \\ & 10 \end{aligned}$ | 2-digit + 2-digit crossing 10s (into 100) <br> See Y2 and now crossing 100s and bridging/carrying $86+48=134$ | 2-digit + 2-digit crossing 10s (into 100) <br> See Y2 and now crossing 100s and bridging/carrying $86+48=134$ <br> Introduce column addition: <br> For $76+47$ $\frac{+47}{\frac{123}{11}}$ | Add <br> Sum <br> More than <br> Total <br> Altogether <br> Plus <br> Partition into hundreds, tens and ones/units <br> Count on <br> Carry ten <br> Bridge ten | 100 square <br> Number lines <br> Number tracks <br> Tens Frame <br> Numicon <br> Place Value Counters <br> Base ten (Dienes) <br> Arrow Cards |



## Year 4 Addition

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabulary | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|c\|} \hline \text { Year } \\ 4 \end{array}$ | Practise mental methods with increasingly large numbers to aid fluency Add numbers mentally, including: A 3-digit number and hundreds <br> A 4-digit number and thousands <br> Add any pair of twodigit numbers, including crossing the tens and 100 boundary, e.g. $47+$ 58 <br> add a near multiple of 10 , e.g. $45+39$ $\begin{gathered} 45+39=84 \\ 45+40-1 \\ 85-\quad 1=84 \end{gathered}$ <br> Add near doubles of two-digit numbers, e.g. $38+37$ | Use of place value counters to add 4 digit numbers and also money too. $\begin{aligned} & \text { (a0) } \\ & 3223+1212=4435 \end{aligned}$ | Use of place value grid. | 4-digit numbers and decimals - same number of digits. <br> Money up to 4 digits $\begin{array}{r} € 38.25 \\ +£ 27.46 \\ \hline \mathbf{€ 6 5 . 7 1} \end{array}$ | Add <br> Sum <br> More than <br> Total <br> Altogether <br> Plus <br> Partition into thousands, hundreds, tens and ones <br> Count on <br> Carry/Bridge ten <br> Carry/Bridge <br> 100 <br> Two digit <br> three digit <br> Four digit <br> Crossing tens boundary | 100 <br> square <br> Number lines <br> Number tracks <br> Place <br> Value <br> Counters <br> Base ten (Dienes). <br> Arrow <br> Cards |



## Year 5 and 6 Addition

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabulary | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year 5 <br> And <br> Year <br> 6 | Y5 <br> Add numbers mentally with increasingly large numbers to aid fluency e.g. $12462+2$ $300=14762$ <br> Use rounding to check answers and determine, levels of accuracy <br> Add a pair of two or threedigit <br> multiples of 10, e.g. $30+80$, $35+36$ and $350+360$ <br> Add a near multiple of 10, 100 | Use of place value counters to add up to 6 digits. | Use of place value grid. See Y4 for some examples. | Varied sized numbers up to millions or 3DP added using compact method. Includes measures and money <br> MAl: Partitioning $\underbrace{4.73+2.21}_{6}=6.94$ | Add <br> Sum <br> More than <br> Total <br> Altogether <br> Plus <br> Partition into hundred thousands, ten thousands, thousands, hundreds, tens and ones Count on <br> Carry ten Carry 100 Carry 1000 Carry 10000 Carry 100000 | 100 square <br> Number lines <br> Number tracks <br> Place Value Counters. <br> Base ten (Diennes). <br> Arrow Cards |


| number or four-digit number, e.g. $235+198$ <br> Add pairs of decimal fractions each with units and tenths, e.g. $5.7+2.5,6.3+4.8$ <br> Y6 <br> Calculate mentally with increasingly large numbers and more complex calculations. Including Counting on in multiples <br> Addition facts for multiples of 10 to 1000 and decimal numbers with one decimal place, |  |  | Decimals - same and different number of digits $\begin{gathered} \text { A7 j: Column Addition } \\ 73.4+5.67=79.07 \\ 7 .+\cdots \\ 7.4 .4 \\ +\frac{5.67}{79.07} \\ \hline \end{gathered}$ | Two digit three digit <br> Crossing tens boundary <br> Inverse <br> addend |  |
| :---: | :---: | :---: | :---: | :---: | :---: |

## Primary Calculation Policy

## EYFS Subtraction




## Year One Subtraction

|  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |



## Year Two Subtraction




Subtracting a 2-digit from a 2-digit number not crossing the tens
$28-13$ - can draw in the place value grid

Can use the bar model, part whole model and number lines as shown above. Part whole below.


Introduce column subtraction wihout regrouping:
$21-10=$
21

- 10

Record the calculation vertically subtracting the column of ones then the column of tens.


## Year 3 Subtraction




## Year 4 Subtraction

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabulary | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|c\|} \hline \text { Year } \\ 4 \end{array}$ | Practise mental methods with increasingly large numbers to aid fluency <br> Subtract any pair of two-digit and threedigit numbers, including crossing the 10 and 100 boundary, e.g. 58 23 <br> Count on and back in 10s from any number <br> Subtract a near multiple of 10, e.g. <br> 84-29 $\square$ <br> Understand subtraction as inverse of addition | Subtracting 4 digit numbers crossing tens and hundreds <br> See Y3 guidance for 3 digit numbers - it is the same principle. <br> Subtracting with money up to 4 digits using decimals <br> Use with real money to show how to find differences | Subtracting 4 digit numbers crossing tens and hundreds <br> See Y3 guidance for 3 digit numbers - it is the same principle. $\square$ $\square$ <br> Subtracting with money up to 4 digits using decimals <br> Children can draw the coins and notes and show the exchange. | Subtracting 4 digit numbers crossing tens and hundreds <br> See Y3 guidance for 3 digit numbers - it is the same principle. <br> Subtracting with money up to 4 digits using decimals $\square$ <br> The ingage part with relationstio ID. ridis1 was not found in the fiec $\square$ | Subtraction <br> Partition into thousands, hundreds, tens and ones <br> Count on <br> Carry back <br> First <br> Then <br> Now <br> Difference <br> Find the difference <br> Decrease / reduced by | 100 square <br> Number lines <br> Number tracks <br> Place <br> Value <br> Counters <br> Base ten <br> Arrow <br> Cards |



## Year 5 and Year 6 Subtraction

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabulary | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year <br> 5 <br> And <br> Year <br> 6 | Subtract numbers with increasingly large numbers to aid fluency e.g. 12 462-2 300= 10162 <br> Use rounding to check answers and determine, levels of accuracy <br> Subtract a pair of two or three-digit multiples of 10, e.g. 80-30, 45-36 and $450-360$ <br> Subtract a near multiple of 10 or 100 from any two-digit or three-digit number, e.g. 235-199 <br> Subtract pairs of decimal fractions each with ones and tenths, $\text { e.g. } 5.7-2.5,6.3-4.8$ | Subtract whole numbers with more than 4 digits and increasingly large numbers using efficient column written methods with decomposition to aid fluency <br> Please see the Year 3 and Year 4 examples as they have the same principles $\qquad$ | Subtract whole numbers with more than 4 digits and increasingly large numbers using efficient column written methods with decomposition to aid fluency <br> Please see the Year 3 and Year 4 examples as they have the same principles <br> The image part with relationship 10 rid151 was not found in the flie. <br> x The image part with relationstip ID ridis1 was not found in the filie. <br> ख The image part with relationship ID ridis1 was not found in the file. | Subtract whole numbers with more than 4 digits and increasingly large numbers using efficient column written methods with decomposition to aid fluency <br> Please see the Year 3 and Year 4 examples as they have the same principles <br> Negative numbers $7-9=-2$ <br> There is a negative difference of 2 <br> The difference between 9 and -3 . $\square$ | Subtraction <br> Partition into millions, hundred thousands, ten thousands, thousands, hundreds, tens and ones <br> Empty number line <br> Count on <br> Carry back <br> First, Then <br> Now <br> Difference <br> Find the difference Decrease / reduced by Negative | 100 square <br> Number lines <br> Number tracks <br> Place Value Count <br> Base ten. <br> Arrow Cards |



## Primary Calculation Policy

## EYFS Multiplication



## Year One Multiplication

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabular y | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year <br> 1 | Count on from and back to zero in ones, twos, fives or tens <br> Make connections between arrays, number patterns, and counting in twos, fives and tens. <br> Recognise odd and even numbers | Repeated Addition - Counting in 2s (also apply to counting in 10's and 5's) <br> Use images of different objects $\square$ <br> There are 7 groups of 2 $\square$ <br> Tens frames can also be used to show times tables such as $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s . | Repeated Addition - - Counting in 2s <br> Draw the objects <br> There are 7 groups of 2 $\square$ <br> Tens frames can also be used to show times tables such as 2 s , 5 s and 10 s . | Repeated Addition - - Counting in 2s <br> Can use bar model, number line and equation $\square$ $\square$ $2+2+2+2+2+2+2=14$ <br> $2 \times 7=14$ (introduce the multiplication symbol) Introduce the multiplication symbol $\begin{aligned} & 5+5+5=5 \times 3= \\ & 15 \end{aligned}$ | Lots of <br> Groups of <br> Times <br> Multiply <br> Repeate d addition <br> Double <br> Sets <br> Groups, <br> Pairs <br> Array | 100 square <br> Number lines <br> Number tracks <br> Bead strings. <br> Tens Frame <br> Numicon <br> Place Value Counters <br> Base ten (Diennes). |




## Year Two Multiplication




## Year 3 Multiplication




## Year 4 Multiplication

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabulary | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|c} \hline \text { Year } \\ 4 \end{array}$ | Recall and practise multiplication facts for tables up to $12 \times$ 12 to aid fluency. <br> Use place value, known and derived facts to multiply mentally, including multiplying by 0 and 1 TU by 4 or 8 , eg. 26 $\times 4$ by doubling three numbers together two digit by a unit eg. $17 \times 3$ numbers to 1000 by 10 and 100 (wholenumber answers) eg. $325 \times 10,42 \times 100$ <br> Extend mental methods to HTU to derive facts e.g. 200 $\times 3=600$ into $600 \div$ $3=200$ <br> Recognise and use factor pairs e.g. give the factor pair associated with a multiplication fact, (if | Consolidate 3, 4, 8 times table | Consolidate 3, 4, 8 times table | Consolidate 3, 4, 8 times table | Lots of | 100 square |
|  |  | See Y3 guidance - same | See Y3 guidance - same | See Y3 guidance - same |  |  |
|  |  | principle | principle | principle | Groups of | Number lines |
|  |  | Learn the remaining tables to | Learn the remaining tables to | Learn the remaining tables to | Times | Number tracks |
|  |  | $\frac{x 12}{\text { See }} \text { Y2 examples showing }$ | $\mathrm{x} 12$ <br> See Y2 examples showing | $\frac{\mathrm{x} 12}{\text { See }} \text { Y3 guidance - same }$ |  |  |
|  |  | concrete groups and arrays as they hold the same principles | pictorial groups and arrays as they hold the same principles | principle | $\begin{array}{\|l\|l\|} \text { Repeated } \\ \text { addition } \end{array}$ | Numicon |
|  |  | Multiply 2 digit by one digit | Multiply 2 digit by one digit |  | Double | Place Value |
|  |  | $24 \times 3$ - Use Dienes or counters | $24 \times 3 \text { - Draw it }$ | Multiply 2 digit by one digit |  | Counters. |
|  |  |  |  |  | Groups, | Base ten (Diennes). |
|  |  | 凹 The image part with relationship id ridi151 was not found in the file. | mammememe |  | Pairs | Arrow Cards |
|  |  |  |  |  | Array | Gattegno chart |
|  |  |  |  |  | symbol x | Place Value |
|  |  |  |  |  | factor | Grid |
|  |  |  | Make connections $\times 10 \times 100$ |  |  |  |
|  |  |  |  | Make connections $\times 10 \times 100$ | multiple |  |
|  |  |  | The counters can be drawn also Using | $12 \times 10=120$ | ten times |  |
|  |  |  |  | $12 \times 100=1200$ | the size |  |
|  |  |  |  |  | hundred |  |



## Year 5 and Year 6 Multiplication

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabulary | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{array}{\|c} \hline \text { Year } \\ 5 \\ \text { and } \\ 6 \end{array}$ | Multiply TU X U mentally using known facts for all multiplication tables to 12 x 12 numbers <br> Identify <br> multiples <br> and factors, <br> including finding <br> all factor pairs <br> for numbers <br> to 100 , e.g. 30 has the factor pairs $1 \times$ <br> $30,2 \times 15,3 \times 10$ and $5 \times 6$ <br> Establish whether a number up to 100 is prime and recall prime numbers up to 19 <br> Recognise and use square and cube numbers, and relevant notation. | Multiply up to 4 digits by a one- number <br> $2214 \times 4$ - use counters $\square$ <br> Multiply up to 4 digits by a two- number <br> If children are working at this level - moving straight to a formal method is the best approach. | Multiply up to 4 digits by a one- number <br> $2214 \times 4$ - can draw in a place value grid $\square$ <br> Multiply up to 4 digits by a two- number <br> If children are working at this level - moving straight to a formal method is the best approach. | Multiply up to 4 digits by a one- number <br> Year 5 <br> Start with expanded with brackets: $\square$ <br> Then move on to compact method showing bridging: $\square$ | As above <br> factor <br> product <br> multiple <br> ten times <br> the size <br> hundred times the size <br> a tenth the size <br> a <br> hundredth the size <br> scaling <br> adjacent multiples <br> prime <br> square <br> cubed | 100 square <br> Number lines <br> Number tracks <br> Tens Frame <br> Numicon <br> Place Value <br> Counters <br> Base ten (Diennes). <br> Arrow Cards <br> Gattegno chart <br> Place Value Grid |



Double decimals with units and tenths, e.g. double 7.6

Scale up and down using known facts, e.g. given that three oranges cost $24 p$, find the cost of four oranges

Identify numbers with an odd number of factors ( square numbers), even numbers of factors and no factor pairs other than 1 and themselves ( prime numbers)

Explore the order of operations using brackets; eg. $2+1 \times 3=5$ and ( $2+1$ ) $\mathrm{x} 3=9$.

Use multiplication facts to solve ratio and proportion problems.

Year 6 - multiply decima
numbers
Start with the expanded method


Then move on to the compact


## EYFS Division

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabulary | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| EYFS | Count in twos; fives; tens both aloud and with objects. | Grouping or Sharing Model | Grouping or Sharing Model Children to draw the concrete resources they are using.$6 \div 3=2$ | Grouping or Sharing Model <br> Write the number sentence | Share <br> Sharing <br> grouping | 100 square |
|  |  | $\square$ I have |  |  |  | Number lines |
|  |  | cubes, |  |  |  | Number tracks |
|  |  | $\qquad$ can you $\square$ share |  |  |  | Bead strings |
|  |  | them equally | $10 \div 2=5$ |  | Groups | Tens Frame |
|  |  | in 2 groups? |  |  | Left over <br> Half | Numicon |
|  |  |  |  |  | Halving | Place Value Counters |
|  |  |  |  |  |  | Base ten (Dienes) |

## Year One Division

|  | Mental Strategies | Concrete | Pictorial | Abstract | $\begin{aligned} & \text { Vocabula } \\ & \text { ry } \end{aligned}$ | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Year } \\ 1 \end{gathered}$ | Share objects into equal groups and count how many in each group and consider 'left over'. <br> Count on from and back to zero in ones, twos, fives or tens - including starting from different points. | Sharing using a range of objects. <br> Focus on EQUAL groups 6 shared by 2 <br> There are 10 sweets. How many people can have 2 sweets each? | Sharing using a range of objects. <br> Focus on EQUAL groups 6 shared by 2 <br> Focus on EQUAL groups Put 8 into groups of 2 <br> $x$ The image part with relationship ID rid151 was not found in the file | Sharing using a range of objects. <br> Focus on EQUAL groups <br> 6 shared by 2 <br> Qmant $\square$ $\square$ <br> Focus on EQUAL groups $\qquad$ <br> Put 8 into groups of 2 <br> $x$ The inage part with relationstip ID rld151 was not found in the file. $\square$ <br> Move on to use a number line to show jumps in groups. The number of jumps equals the number of groups. | Share <br> Sharing <br> grouping <br> Equal <br> Groups <br> Left over <br> Half <br> Halving | 100 square <br> Number lines <br> Number tracks <br> Bead strings <br> Tens Frame <br> Numicon <br> Place Value Counters <br> Base ten (Dienes) |

## Year Two Division

|  | Mental Strategies | Concrete | Pictorial | Abstract | Vocabula ry | Models, Images and resources |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year 2 | Practise to become fluent in recall and use of multiplication and division facts for the 2,5 and 10 multiplication tables, <br> Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot <br> Introduce the symbol for division - | Sharing into arrays of 2,5 and 10 using a range of objects <br> Link division to multiplication by creating an array and thinking about the number sentences that can be created. $\begin{array}{rlr} \text { Eg } 15 \div 3=5 & 5 \times 3=15 \\ 15 \div 5=3 & 3 \times 5=15 \end{array}$ $\square$ <br> x The image part with relationship LD rid151 was not found in the file. | Sharing into arrays of 2,5 and 10 using a range of objects <br> Children may begin with less sophisticated drawings of groupings $30 \div 5=6$ $\square$ <br> And move on to draw an array showing groups to make multiplication and division sentences. $\begin{aligned} & 15 \div 5=3 \\ & 15 \div 3=5 \end{aligned}$ $\square$ | Sharing into arrays of 2,5 and 10 using a range of objects (using only numerals) $30 \div 5=6$ $\square$ $40 \div 10=4$ <br> On a number line they could jump forwards or backwards. $\square$ <br> Also link to inverse number $\begin{aligned} & 40 \div 10=4 \\ & 40 \div 4=10 \\ & 4 \times 10=40 \\ & 10 \times 4=40 \end{aligned}$ | Divide <br> Share equally <br> One each, two each..., <br> Grouping <br> Equal groups <br> How <br> many <br> lots of.... <br> How <br> many groups of... <br> half of <br> halved <br> symbol $\div$ | 100 square <br> Number lines <br> Number tracks <br> Bead strings <br> Tens Frame <br> Numicon <br> Place Value Counters <br> Base ten (Dienes) |



## Year 3 Division



| $\begin{aligned} & 60 \div 3=20 \text { and } 20 \\ & =60 \div 3 \end{aligned}$ <br> Divide TU and HTU numbers by 10 , understand the effect of $\div 10$ e.g. $700 \div 10$, $\square$ <br> Also use the Gattegno Chart to help <br> Identify remainders when dividing by 2 , 5 or 10 | 2 digit divided by 1 digit with remainders (using Y3 ARE times tables) $87 \div 4=21 \mathrm{r} 3$ |  | 2 digit divided by 1 digit with remainders (using Y3 ARE times tables) $92 \div 10=3 \text { r2 }$ | a tenth of the size |  |
| :---: | :---: | :---: | :---: | :---: | :---: |

## Year 4 and Year 5 Division




## Year 6 Division



|  |  | Use long division to divide any sized number by a 2diigit number (including remainders) <br> No concrete representation due to complexity. | Use long division to divide any sized number by a 2-diigit number (including remainders) No pictorial representation due to complexity. | Also apply to decimal numbers Year 6 - Finally move into decimal places to divide the total accurately. $79 \div 4=19.75$ $\text { 4) } \frac{19.75}{7^{3} 9.0^{3} 0}$ <br> $39.9 \div 7=5.7$ <br> 7) $\frac{05.7}{3^{3} 9 .{ }^{4} 9}$ <br> Use long division to divide any sized number by a 2diigit number (including remainders) <br> Staff to have flexibility between use of chunking method and use of arrows (each to be used alongside a fact box) | a tenth or hundredth of the size <br> Fraction <br> Decimals |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |



## Glossary

2-digit - a number with 2 digits like 23, 45, 12 or 60
3-digit - a number with 3 digits like 123, 542, 903 or 561
Addition facts - knowing that $1+1=2$ and $1+3=4$ and $2+5=7$. Normally we only talk about number facts with totals of 20 and under.
Array -An array is an arrangement of a set of numbers or objects in rows and columns -it is mostly used to show how you can group objects for repeated addition or subtraction.

Bridge to ten - a strategy when using number lines. Adding a number that takes you to the next 'tens' number.
Concrete apparatus - objects to help children count - these are most often cubes (multilink) but can be anything they can hold and move.
Dienes (purple hundreds, tens and units blocks), Base Ten, Numicon, Cuisenaire rods are also referred to as concrete apparatus.
Column chunking - method of division involving taking chunks or groups or the divisor away from the larger number
Decimal number - a number with a decimal point
Divisor - the smaller number in a division calculation. The number in each group for chunking.
Double - multiply a number by 2
Exchanging - Moving a 'ten' or a 'hundred' from its column into the next column and splitting it up into ten 'ones' (or 'units') or ten 'tens' and putting it into a different column

Expanded Multiplication - a method for multiplication where each stage is written down and then added up at the end in a column
Find the difference - A method for subtraction involving counting up from the smaller to the larger number
Half - a number, shape or quantity divided into 2 equal parts Halve - divide a number by 2
Integer - a whole number with no decimal point
Inverse - the opposite operation. Addition is the inverse of subtraction, multiplication is the inverse of division
Long Multiplication - column multiplication where only the significant figures are noted

Number bonds to ten -2 numbers that add together to make ten, like 2 and 8 , or 6 and 4 .
Number bonds to $\mathbf{1 0 0}-2$ numbers that add together to make 100 like 20 and 80 , or 45 and 55 or 12 and 88
Number line - a line either with numbers or without (a blank number line). Children use this tool to help them count on for addition of subtraction and also in multiplication and division.

Number sentence - writing out a calculation with just the numbers in a line E.G. $2+4=6$ or $35 \div 7=5$ or $12 \times 3=36$ or $32-5=27$
Partition - split up a larger number into the hundreds, tens and units. E.G. 342 - 300 and 40 and 2
Place Value - knowing that in the number 342 - the ' 3 ' means ' 3 hundreds', the ' 4 ' means ' 4 tens' and the ' 2 ' means ' 2 '.
Quarter - a number, shape or quantity divided into 4 equal parts Remainder - a whole number left over after a division calculation repeated addition repeatedly adding groups of the same size for multiplication

Short division Method - traditional method for division with a single digit divisor.
Significant digit - the digit in a number with the largest value. E.G in 34 - the most significant digit is the 3 , as it has a value of ' 30 ' and the ' 4 ' only has a value of '4'

Single digit - a number with only one digit. These are always less than 10.
Tens number - a number in the ten times tables - 10,20,30,40 50, etc.
Ones - another term for single digit numbers. The right hand column in column methods is the 'ones' column

## Progression of Additional Vocabulary:

- Addition:
- Add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, sum, partition, column, tens boundary, hundreds boundary, increase, vertical, bridging, expanded, compact, inverse, thousands, hundreds, digits, decimal point and decimal
- Subtraction:
- Equal to, less, minus, subtract, distance between, difference, number line, how many more, how many fewer, less than, most, least, count back, how many left, how much less is, count on, difference, count on, strategy, exchange, decrease, value, inverse, decimal point, decimal, tenths and hundredths.
- Multiplication:
- Groups of, times, multiply, count, array, altogether, multiplied by, repeated addition, column, row, sets of, commutative, equal groups, as big as, one twice three times, partition, grid method, multiple, product, tens, units, lots of, equal groups, square, factor, integer, decimal, short/long, carry and decimal.
- Division
- Share, share equally, one each, two each, group, groups of, lots of, array, divide, divided by, divided into, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, factor, divisible by, common factor, remainder, quotient, prime number, prime factors and composite number (non-prime)


## Appendix

Listed below are a range of recommendations and teaching ideas aimed at informing and enhancing the teaching of primary mathematics:

## 1. Developing children's understanding of the $=$ symbol

The $=$ symbol is an assertion of equivalence. If we write $3+4=6+1$ then we are saying that what is on the left of the $=$ symbol is equivalent to what is on the right of the symbol. But many children interpret = as always being an instruction to work out the value of a calculation. This is as a result of always seeing it used as follows:
$3+4=$
$5 \times 7=$
16-9 =
If children only think of = as meaning "Work out the answer to this calculation" then they are likely to get confused by empty box questions such as:
$3+\square$ $=8$ and are are very likely to struggle with even simple algebraic equations, such as: $3 y=18$. This can be overcome by doing the following:

- Vary the position of the $=$ symbol e.g. $24=4 \times 6$
- Include lots of empty box problems e.g. 12 -4;$x 6=24$
- Teach inequality alongside equality e.g. $5+9$$3 \times 5$ (< > or =?)


## 2. <br> Recognising the actual value of ones, tens, hundreds etc. in a number

Many children are able to recognise the value of each digit in a number like 347 but find it harder to explain, for example, how many tens there are in 347 . Once they are able to recognise that there are 34 tens (rather than 4 tens), it makes it much easier to be able to carry out a calculation such as $347+30$ as they are adding 3 tens to the 34 tens. Traditionally, children often struggle when tackling a calculation involving crossing over a hundred e.g. $293+10$ but using this method takes much of the difficulty away as they only need to add 1 ten to the 29 tens to give 30 tens and an answer of 303 . It is equally effective when subtracting e.g. for $112-20$, we subtract 2 tens from the 11 tens to leave is with 9 tens and an answer of 92 .

## 3. Reasoning about mathematical relationships

Children need to be exposed to images and structures that help them to make links between inverse operations from an early age


Opportunities should be taken wherever possible to demonstrate how children can use what they already know to work out a related fact e.g.:
.- if $6+4=10$, then 6 tens +4 tens $=10$ tens i.e. $60+40=100$

- If you know $3+5$, you can use this to work out $23+5$


## 4. Developing children's fluency with basic number facts

Fluent computational skills are dependent on accurate and rapid recall of basic number bonds to 20 and times-tables facts. Research has shown that spending a short time every day on these basic facts quickly leads to improved fluency.

## 5. Developing fluency in mental calculations (The Magic 10)

Children who learn to 'make 10 ' to create an easier calculation are able to develop mental fluency and an ability to look for patterns. Using knowledge of number bonds that make 10 , they can see that $9+6=9+1+5=10+5=15$

| Addition | Regroup $9+3$ into $10+2$ before adding together: $\square$ <br> $6+5=11$ <br> Start with the bigger number and use the smaller number to make 10 . | $x$ The image part with relationship ID rid151 was not found in the file.  <br>   <br>   <br>   <br>   <br> Use pictures or a number line. Regroup or partition the smaller number $\boxed{x}$ The image part with relationship ID rid151 was not found in the file. <br> to make 10 before adding. <br> Children move on to using an 'empty number line'. E.g. $7+5$ becomes $7+3+2$ | $7+5=7+3+2=12$ <br> If I have seven, how many of my 5 do I need to add to make 10. How many more do I still need to add on? |
| :---: | :---: | :---: | :---: |
| Subtraction | $14-5=$ |  |  |
|  |  | Start at 13. Count back 3 to reach 10. Then count | $16-8=$ <br> How many do we take off to reach the previous 10? (6) |
|  | away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9 . | back the remaining 4 so you have taken away 7 altogether. You have reached your answer. | How many do we have left to take off? (2) |

